

# Package ‘picreg’

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**Type** Package

**Title** Variable Selection using the Pivotal Information Criterion

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**Description** Sparse regression and classification via the Pivotal Information Criterion (PIC), an alternative to the Bayesian Information Criterion (BIC), cross-validation, and Lasso-based tuning. The regularisation parameter is selected from a pivotal null-distribution statistic, eliminating the need for cross-validation and yielding sharper support recovery. Provides Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) optimisation for the L1, Smoothly Clipped Absolute Deviation (SCAD), and Minimax Concave Penalty (MCP) penalties across six response distributions: Gaussian, binomial, Poisson, exponential, Gumbel, and Cox. Under standard sparsity assumptions, the selector achieves a phase transition for exact support recovery, analogous to results in compressed sensing. See Sardy, van Cutsem and van de Geer (2026) <[doi:10.48550/arXiv.2603.04172](https://doi.org/10.48550/arXiv.2603.04172)>.

**License** GPL-2

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**30**

assess *Assess performance of a pic fit.*

### Description

Given a test set, reports a small set of family-appropriate predictive metrics. Optionally appends support-recovery diagnostics when the true active set is known.

### Usage

```
assess(object, ...)

## S3 method for class 'pic'
assess(object, newx, newy, true_features = NULL, ...)
```

### Arguments

object	A fitted pic object.
...	Unused; present for S3 method consistency.
newx	Numeric design matrix at which predictions are evaluated, with the same columns as the training data.
newy	Response on the new observations. Numeric vector for all families except Cox; for Cox, a two-column matrix (time, event) matching the training response format.
true_features	Optional integer or character vector listing the indices (or names) of the true active variables. When supplied, support-recovery metrics are appended.

### Details

The metrics depend on the family:

"gaussian" MSE, MAE, R-squared.

"binomial" accuracy, AUC, binomial deviance.

"poisson" MSE, MAE, Poisson deviance.

"exponential" MSE, MAE, Exponential deviance.

"gumbel" MSE, MAE, deviance computed from the per-sample negative log-likelihood with a moment estimate of the scale parameter  $\hat{\sigma} = \text{sd}(y - \hat{\eta})\sqrt{6/\pi}$ .

"cox" Harrell's C-index and the Breslow partial log-likelihood (negative, normalised by n).

When true\_features is non-NULL, four support-recovery metrics are appended (independent of newx / newy): exact\_recovery, tpr (true-positive rate / sensitivity), fdr (false-discovery rate) and f1 (harmonic mean of precision and recall). Names are accepted in addition to integer positions when the fit carries column names.

**Value**

A two-column data.frame with columns metric (character) and value (numeric).

**Examples**

```
data(QuickStartExample)
X <- QuickStartExample$X
y <- QuickStartExample$y
fit <- pic(X, y, family = "gaussian", penalty = "scad")
assess(fit, X, y, true_features = paste0("gene_", 1:5))
```

---

baseline_functions	<i>Breslow baseline cumulative hazard and survival.</i>
--------------------	---

---

**Description**

Breslow baseline cumulative hazard and survival.

**Usage**

```
baseline_functions(times, events, predictions)
```

**Arguments**

times	Survival times (length n).
events	Event indicators (0 or 1; length n).
predictions	Risk scores; higher = higher risk = shorter survival.

**Value**

A data.frame with columns time, cumulative\_hazard, survival.

---

BinomialExample	<i>Small binary-classification dataset for the Binomial section of the vignette.</i>
-----------------	--

---

**Description**

A synthetic logistic-regression dataset used to illustrate pic() with the Binomial family. It contains  $n = 300$  observations of  $p = 50$  predictors, of which  $s = 5$  carry signal; the remaining 45 are noise.

**Usage**

```
BinomialExample
```

**Format**

A list with two components:

x Numeric matrix of dimension  $300 \times 50$  with column names `gene_*` and `noise_*`.

y Integer vector of length 300 containing 0/1 class labels.

**Details**

Column names follow the same convention as [QuickStartExample](#): active variables are labelled `gene_1, ..., gene_5` and inactive ones `noise_1, ..., noise_45`, interleaved in random order.

The binary response is generated as

$$Y_i \sim \text{Bernoulli}\left(\frac{1}{1 + e^{-X_i\beta}}\right),$$

with non-zero coefficients drawn uniformly in  $[1.5, 3]$  with random sign. The class balance is roughly 45% of positives.

**Examples**

```
data(BinomialExample)
fit <- pic(BinomialExample$X, BinomialExample$y, family = "binomial")
fit$selected
```

---

 coef.pic

---

*Coefficients of a fitted pic model.*


---

**Description**

Returns a two-column data frame with the variable name in the first column (`variable`) and the fitted coefficient in the second (`coefficient`). The first row is always the intercept, labelled "(Intercept)"; when no intercept was fitted its value is  $\emptyset$ . Variable names are taken from the column names of `X` if any were supplied (matrix `colnames(X)` or data-frame column names), and otherwise default to `V1, ..., Vp`.

**Usage**

```
## S3 method for class 'pic'
coef(object, standardized = FALSE, ...)
```

**Arguments**

<code>object</code>	A fitted pic object.
<code>standardized</code>	Logical; if TRUE, return the coefficients on the standardised scale used internally during fitting. Default FALSE (return coefficients on the original scale of <code>X</code> ).
<code>...</code>	Unused; present for S3 method consistency.

**Details**

Internally the model is fitted on a standardised design matrix, so the raw coefficients live on the standardised scale. By default this method rescales them back to the **original scale** of  $X$  — the values to plug into the un-standardised design for prediction — via

$$beta\_orig = beta/s \quad intercept\_orig = intercept - sum(m * beta\_orig)$$

where  $m$  and  $s$  are the column mean and standard deviation. Pass `standardized = TRUE` to skip the rescaling and return the raw fit values (identical to `fit$beta`).

**Value**

A `data.frame` with columns `variable` (character) and `coefficient` (numeric), of length  $p + 1$  (intercept + features).

---

concordance_index	<i>Harrell's concordance index.</i>
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---

**Description**

Harrell's concordance index.

**Usage**

```
concordance_index(times, events, predictions)
```

**Arguments**

<code>times</code>	Survival times (length $n$ ).
<code>events</code>	Event indicators (0 or 1; length $n$ ).
<code>predictions</code>	Risk scores; higher = higher risk = shorter survival.

**Value**

Numeric scalar in  $[0, 1]$ .

---

CoxExample

*Small survival dataset for the Cox section of the vignette.*


---

**Description**

A synthetic survival dataset used to illustrate `pic()` with the Cox family. It contains  $n = 250$  subjects observed on  $p = 50$  covariates, of which  $s = 5$  carry signal; the remaining 45 are noise.

**Usage**

```
CoxExample
```

**Format**

A list with two components:

`X` Numeric matrix of dimension  $250 \times 50$  with column names `gene_*` and `noise_*`.

`y` Numeric matrix of dimension  $250 \times 2$  with columns `time` and `event`.

**Details**

Column names follow the same convention as [QuickStartExample](#): active variables are labelled `gene_1`, `...`, `gene_5` and inactive ones `noise_1`, `...`, `noise_45`, interleaved in random order.

Event times are drawn from an exponential proportional-hazards model

$$T_i \sim \text{Exp}(e^{X_i\beta}),$$

and independent censoring times from  $C_i \sim \text{Exp}(0.5)$ . The observed response is the standard two-column  $(\min(T_i, C_i), \mathbf{1}\{T_i \leq C_i\})$ . The censoring rate is roughly 40%.

**Examples**

```
data(CoxExample)
fit <- pic(CoxExample$X, CoxExample$y, family = "cox")
fit$selected
```

---

`cox_partial_log_likelihood`
*Breslow partial log-likelihood (negative, normalised by n).*


---

**Description**

Breslow partial log-likelihood (negative, normalised by n).

**Usage**

```
cox_partial_log_likelihood(times, events, predictions)
```

**Arguments**

times	Survival times (length n).
events	Event indicators (0 or 1; length n).
predictions	Risk scores; higher = higher risk = shorter survival.

**Value**

A numeric scalar: the negative Breslow partial log-likelihood for the Cox proportional-hazards model, normalised by the sample size n. Lower values indicate a better fit.

---

feature\_effects\_on\_survival

*Effect of one feature on the Cox survival curve.*

---

**Description**

Builds the family of survival curves obtained by varying a single covariate while holding the others at their column means. The returned object has the same shape as `predict_survival_function()`, so it composes directly with `plot_survival_curves()`.

**Usage**

```
feature_effects_on_survival(object, idx, values = NULL)
```

**Arguments**

object	A fitted <code>pic.cox</code> object.
idx	Index of the feature to vary. Either an integer column position in the training X or, if X carried column names, the variable name. The feature must lie in the model's selected support; otherwise the curve would be flat in v and the call is rejected.
values	Optional numeric vector of values to evaluate. When NULL (default), the cached grid described in <i>Details</i> is used.

**Details**

Both the per-column mean row and a default grid of representative values (used when `values = NULL`) are cached on the fit by `pic()` under `attr("fit", "preproc")`, so the training design matrix does not need to be passed back in. The cached default grid uses the unique values of the column when there are at most five of them (handy for ordinal / categorical covariates), and the four equispaced empirical quantiles (0, 1/3, 2/3, 1) otherwise.

**Value**

A list with components `time` (length K) and `survival` (matrix K × length(values), one column per evaluated value). Column names of `survival` are formatted as "`<feature_name> = <value>`" and are picked up automatically by `plot_survival_curves()` for the legend.

**See Also**

[predict\\_survival\\_function\(\)](#), [plot\\_survival\\_curves\(\)](#).

---

lambda\_pdb

*Pivotal Detection Boundary regularisation selector*

---

**Description**

Computes the data-driven regularisation parameter  $\hat{\lambda}_\alpha^{\text{PDB}}$  using the Pivotal Detection Boundary (PDB) principle. The selected value is defined as the empirical  $(1-\alpha)$  quantile of a null-distribution gradient statistic

$$\hat{\lambda}_\alpha^{\text{PDB}} = q_{1-\alpha}(\|\nabla\ell_0\|_\infty),$$

where  $\ell_0$  denotes the loss evaluated under the null model.

**Usage**

```
lambda_pdb(
  X,
  family,
  n_simu = 5000L,
  alpha = 0.05,
  method = c("mc_exact", "mc_gaussian", "analytical")
)
```

**Arguments**

X	Numeric design matrix. Columns should typically be standardised to zero mean and unit variance.
family	A PIC family specification. Can be either a family object or a character string accepted by <a href="#">get_family()</a> .
n_simu	Number of Monte Carlo simulations used by the stochastic methods. Ignored when method = "analytical".
alpha	Nominal tail probability used to define the quantile level.
method	One of "mc_exact", "mc_gaussian", or "analytical".

**Details**

Under the null  $\beta = 0$ , the gradient of the loss carries only sampling noise. The smallest  $\lambda$  large enough to dominate this noise is the natural threshold separating signal from noise, i.e. the value above which a coefficient should be kept rather than shrunk to zero. Calibrating  $\hat{\lambda}$  this way has two consequences. First, the quantile depends only on the design matrix  $X$ , the family, and the level  $\alpha$  (not on the response  $y$ ), so cross-validation is no longer needed. Second, and more importantly, it leads to sharper support recovery than prediction-error-based selectors such as cross-validated lasso: by targeting the noise level of the gradient directly, PDB controls the inclusion of noise variables and more reliably identifies the true non-zero coefficients. Computing the quantile requires only the distribution of  $\|\nabla\ell_0\|_\infty$ , which `lambda_pdb()` estimates through one of the three methods below.

**Details on method option:**

The empirical quantile is obtained using one of the three following methods:

"mc\_exact" Family-aware Monte Carlo. For each of the  $n_{\text{simu}}$  draws, a response vector is sampled under the null model ( $\beta = 0$ ) of the chosen family; the family-specific gradient of the loss at  $\beta = 0$  is evaluated on the fixed design  $X$ , and its supremum norm is recorded. The empirical  $(1 - \alpha)$  quantile of the  $n_{\text{simu}}$  recorded norms is returned. Most accurate but slowest of the three.

"mc\_gaussian" Monte Carlo under the Gaussian approximation of the null gradient. A central-limit argument gives  $\nabla \ell_0 \approx \mathcal{N}(0, c(n) \Sigma_X/n)$  with  $\Sigma_X = X^\top X/n$ . Each of the  $n_{\text{simu}}$  draws samples directly from this Gaussian and records its supremum norm — no family-specific evaluation needed. Family-agnostic and noticeably faster than "mc\_exact"; valid in the regime where the CLT kicks in (moderate to large  $n$ ).

"analytical" Closed-form Bonferroni bound on the Gaussian tail. Combining a union bound over the  $p$  coordinates of the gradient with the standard Gaussian tail bound gives

$$\hat{\lambda}_\alpha^{\text{analytical}} = \Phi^{-1}(1 - \alpha/(2p)) \sqrt{c(n)/n}.$$

Deterministic and  $O(1)$  — no simulation. Conservative (slightly over-estimates the true quantile) and gets looser as  $p$  grows, but useful when speed matters or when Monte Carlo is overkill.

The "mc\_gaussian" and "analytical" methods use a variance scaling factor  $c(n)$  depending on the family:

- Gaussian / Binomial / Poisson / Exponential:  $c(n) = 1$
- Gumbel:  $c(n) = \exp(2(\gamma + 1))$ , with  $\gamma$  the Euler-Mascheroni constant.
- Cox:  $c(n) = 1/(4 \log n)$

**Computational cost:**

Both Monte Carlo methods are dominated by a single  $p \times n_{\text{simu}}$  matrix product  $X^\top R$ , where  $R$  stacks the simulated residuals. This product is dispatched to BLAS as a single gemm, which is essentially as fast as a Monte Carlo selector can be on dense designs. For very large  $n$  or  $p$ , however, the constant becomes large and "mc\_exact" may be unnecessarily expensive:

- As  $n$  grows, the central-limit approximation tightens and "mc\_gaussian" gives essentially the same  $\hat{\lambda}$  as "mc\_exact" at a fraction of the cost (no family-specific residual generation, fewer dependencies on  $y$ -draws).
- "analytical" is  $O(1)$  and useful as a quick upper bound — slightly conservative, but accurate enough for triage and for very high  $p$  where the Bonferroni tail is tight.

Practical rule of thumb: prefer "mc\_exact" for small to moderate problems (default), "mc\_gaussian" once  $n$  grows and the design is dense, and "analytical" when even the Monte Carlo cost is a concern.

**Details on alpha option:**

The level  $\alpha$  is the nominal Type-I error of the test of  $H_0: \beta = 0$ . By construction of the quantile,

$$\Pr\left(\|\nabla \ell_0\|_\infty > \hat{\lambda}_\alpha^{\text{PDB}} \mid H_0\right) = \alpha,$$

so under the null model no variable enters the active set with probability at most  $\alpha$ . With the default  $\alpha = 0.05$ , this caps the false-discovery rate at 5% under the null: when the data carry no signal, picreg returns the empty support 95% of the time.

**Value**

An object of class "pic.lambda\_pdb".

value	Selected regularisation parameter $\hat{\lambda}_\alpha^{\text{PDB}}$ .
statistics	Simulated null statistics used to estimate the quantile. NULL for the analytical method.
method	Estimation method used.
alpha	Quantile level parameter.
n_simu	Number of Monte Carlo simulations.

**Examples**

```
X <- scale(matrix(rnorm(100 * 20), 100, 20))
lam <- lambda_pdb(
  X,
  family = "gaussian",
  method = "mc_exact"
)
print(lam)
```

---

pdb\_asymptotic

*Asymptotic behaviour of the PDB null distribution.*

---

**Description**

For each  $n$  in  $n\_grid$ , draws a standardised Gaussian design matrix of shape  $(n, p)$  and computes the null gradient-norm statistic via the three available selectors: "mc\_exact", "mc\_gaussian", and "analytical". Stores the simulated Monte Carlo statistics and the three resulting  $\hat{\lambda}$  values per  $n$ .

**Usage**

```
pdb_asymptotic(
  n_grid,
  p,
  type = c("gaussian", "binomial", "poisson", "exponential", "gumbel", "cox"),
  alpha = 0.05,
  n_simu = 5000L,
  verbose = FALSE
)
```

**Arguments**

n_grid	Integer vector of sample sizes to evaluate.
p	Number of features (scalar integer).

type	Family name: "gaussian", "binomial", "poisson", "exponential", "gumbel", or "cox".
alpha	Nominal level used for the (1 - alpha) quantile.
n_simu	Monte Carlo size for each selector.
verbose	Logical; if TRUE, prints a one-line progress message per n.

### Details

The intended use is to **visualise the convergence** of the exact family-specific null distribution to the Gaussian approximation as  $n$  grows — i.e., to check empirically that `mc_gaussian` is a valid substitute for `mc_exact` in the asymptotic regime.

### Value

An object of class `c("pic.pdb_asymptotic", "pic.diagnostic")`.

`n_grid`, `p`, `type`, `alpha`, `n_simu`

Configuration.

`stats_exact`, `stats_gaussian`

Lists of length `length(n_grid)` where each element is a numeric vector of length `n_simu` containing the simulated null statistics from the corresponding selector.

`lambda_exact`, `lambda_gaussian`, `lambda_analytical`

Numeric vectors of length `length(n_grid)` - the (1 - alpha) quantile under each selector at each  $n$ .

`call` The call.

### See Also

`plot.pic.pdb_asymptotic()` for visualisation.

### Examples

```
as_ <- pdb_asymptotic(n_grid = c(50, 200, 1000),
                    p = 200, type = "poisson")
plot(as_)
```

### Description

Prints a formatted summary of the Pivotal Detection Boundary selector used by `pic()` to choose  $\hat{\lambda}$ : method, nominal level, Monte Carlo size, selected  $\hat{\lambda}$ , and a compact view of the null distribution when Monte Carlo was run. For models fitted with `method = "analytical"` or with a user-supplied `lambda`, only the selector metadata is shown.

**Usage**

```
pdb_summary(model, digits = 4L)
```

**Arguments**

```
model          A fitted pic object.
digits         Number of significant digits used in the distribution table.
```

**Value**

Invisibly returns NULL. Called for its side effect (printing).

**Examples**

```
data(QuickStartExample)
fit <- pic(QuickStartExample$X, QuickStartExample$y,
          family = "gaussian", penalty = "lasso")
pdb_summary(fit)
```

---

phase_transition	<i>Phase-transition analysis of support recovery.</i>
------------------	---

---

**Description**

For a fixed  $(n, p)$  configuration, varies the sparsity level  $s$  from  $0$  to  $s_{\max}$  and, for each  $s$ , estimates by Monte Carlo ( $m$  replications) the probability that `pic()` recovers exactly the support of the true coefficient vector. If several penalties are supplied, one result is produced for each  $(n, p, \text{penalty})$  combination.

**Usage**

```
phase_transition(
  n,
  p,
  type = c("gaussian", "binomial", "poisson", "exponential", "gumbel", "cox"),
  s_max,
  m = 50,
  penalty = "lasso",
  beta_value = 3,
  lambda_method = "mc_exact",
  lambda_alpha = 0.05,
  lambda_n_simu = 5000L,
  verbose = FALSE,
  parallel = FALSE,
  workers = parallel::detectCores() - 1L
)
```

**Arguments**

n	Integer or integer vector — number of observations per configuration.
p	Integer or integer vector of the same length as n — number of features.
type	Family name: "gaussian", "binomial", "poisson", "exponential", "gumbel", or "cox".
s_max	Largest sparsity level evaluated. Must satisfy $s\_max < \min(p)$ .
m	Number of Monte Carlo replications per s.
penalty	One or more penalties for pic(): "lasso", "scad", or "mcp".
beta_value	Magnitude of the non-zero coefficients used to generate the response. The sign is fixed to +.
lambda_method	Passed to pic(). Default "mc_exact".
lambda_alpha	Nominal level for the PDB selector.
lambda_n_simu	Monte Carlo size for the PDB selector.
verbose	Logical; if TRUE, prints a one-line progress message per (n, p, penalty, s).
parallel	Logical; if TRUE, distribute the m Monte Carlo replications of each (n, p, penalty, s) cell across multiple R processes via the future framework (future::multisession plan).
workers	Integer; number of background R processes to use when parallel = TRUE. Ignored otherwise.

**Details**

At each Monte Carlo replicate a design matrix is drawn from a standard Gaussian, s features are sampled uniformly at random, the response is generated under the chosen type, and one pic fit is run for each requested penalty. The selected support is compared to the truth and three metrics are stored:

exact\_recovery 1 if the selected set equals the true set, 0 otherwise.

tpr  $|\hat{S} \cap S| / |S|$  - true positive rate. For  $s = \emptyset$ , this is set to 1 when no true feature exists.

fdr  $|\hat{S} \setminus S| / \max(|\hat{S}|, 1)$  - false discovery rate.

**Details on data sampling:**

At each replicate, the design  $X$  is drawn iid  $\mathcal{N}(0, 1)$ , the true support  $S$  is sampled uniformly at random, and  $\beta_j = \text{beta\_value}$  for  $j \in S$ ,  $\beta_j = 0$  otherwise. The linear predictor is  $\eta = X\beta$ . The response  $y$  is then drawn conditionally on  $\eta$  according to the requested family:

"gaussian"  $y_i = \eta_i + \varepsilon_i$  with  $\varepsilon_i \sim \mathcal{N}(0, 1)$ .

"binomial"  $y_i \sim \text{Bernoulli}(\sigma(\eta_i))$ , where  $\sigma(z) = 1/(1 + e^{-z})$  is the logistic function.

"poisson"  $y_i \sim \text{Poisson}(e^{\eta_i})$ .

"exponential"  $y_i \sim \text{Exp}(\text{rate} = e^{\eta_i})$ .

"gumbel"  $y_i = \eta_i + \varepsilon_i$  with  $\varepsilon_i \sim \text{Gumbel}(0, 1)$ , drawn as  $-\log(-\log U_i)$  for  $U_i \sim \mathcal{U}(0, 1)$ .

"cox" Event times  $T_i \sim \text{Exp}(e^{\eta_i})$  and independent censoring times  $C_i \sim \text{Exp}(1)$ . The response is the 2-column matrix  $(\min(T_i, C_i), \mathbf{1}\{T_i \leq C_i\})$ .

**Value**

An object of class `c("pic.phase_transition", "pic.diagnostic")`.

`s_grid`            `0:s_max`.  
`exact_recovery, tpr, fdr`  
                     Matrices of shape  $(\text{length}(n) * \text{length}(\text{penalty}), s\_max + 1)$  - one row per  $(n, p, \text{penalty})$  curve, one column per sparsity level  
`curve_n, curve_p, curve_penalty`  
                     Curve descriptors aligned with the rows of the metric matrices.  
`config`            A list of all configuration arguments for downstream plotting / reporting.  
`call`              The call.

**See Also**

[plot.pic.phase\\_transition\(\)](#) for visualisation.

**Examples**

```
pt <- phase_transition(n = 50, p = 100, type = "gaussian",
                     s_max = 8, m = 20,
                     penalty = c("lasso", "scad"),
                     parallel = TRUE)

plot(pt)
```

---

pic

*Sparse regression via pivotal loss and PDB lambda selection*


---

**Description**

Fits sparse regression, classification, and survival models using a family-specific pivotal loss combined with a sparsity-inducing penalty. Supported families are Gaussian, binomial, Poisson, exponential, Gumbel, and Cox proportional hazards.

**Usage**

```
pic(
  X,
  y,
  family = c("gaussian", "binomial", "poisson", "exponential", "gumbel", "cox"),
  penalty = "lasso",
  standardize = TRUE,
  intercept = TRUE,
  lambda_method = c("mc_exact", "mc_gaussian", "analytical"),
  relax = FALSE,
  mcp_gamma = 3,
  scad_a = 3.7,
```

```

lambda = NULL,
lambda_alpha = 0.05,
lambda_n_simu = 2000,
tol = 1e-05,
path_length = 10L,
maxit = 1000
)

```

## Arguments

<code>X</code>	Numeric design matrix of shape $(n, p)$ , where rows are observations and columns are candidate predictors. <code>X</code> may be a matrix or a data frame coercible to a numeric matrix. It must contain at least two columns and no NA, NaN, or infinite values.
<code>y</code>	Response variable with length $n$ . For Gaussian and Gumbel models, <code>y</code> must be numeric. For Binomial models, <code>y</code> must contain only 0 and 1. For Poisson and Exponential models, <code>y</code> must be non-negative. For <code>family = "cox"</code> , <code>y</code> must be a two-column numeric matrix ( <code>time</code> , <code>event</code> ), where <code>time</code> is non-negative and <code>event</code> is coded as 0 or 1.
<code>family</code>	A character name determining the response distribution and the associated loss function used during optimisation. See <b>Details on family-specific losses</b> below for the exact objective functions associated with each family. Default <code>"gaussian"</code> .
<code>penalty</code>	Penalty name: one of <code>"lasso"</code> , <code>"scad"</code> , or <code>"mcp"</code> (lowercase). Default <code>"lasso"</code> . See <a href="#">pic_penalties</a> for the precise form of each penalty and the role of <code>scad_a / mcp_gamma</code> .
<code>standardize</code>	Whether to standardise columns of <code>X</code> to zero mean and unit variance prior to computing the PDB regularisation parameter and fitting the model. Strongly recommended for proper PDB calibration. When TRUE, the optimisation is performed on the standardised design matrix and the stored fitted coefficients therefore correspond to the standardised scale. Use <code>coef.pic()</code> to recover coefficients on the original scale of <code>X</code> . Default is TRUE. Standardisation centres <code>X</code> , so for non-Cox families it can only be combined with <code>intercept = TRUE</code> (the intercept absorbs the column-mean shift); pass <code>standardize = FALSE</code> if you really want <code>intercept = FALSE</code> .
<code>intercept</code>	Whether to fit an unpenalised intercept. Forced to FALSE for the Cox family (whose partial likelihood is invariant under translation of the linear predictor, so centring of <code>X</code> is harmless). For other families, <code>intercept = FALSE</code> requires <code>standardize = FALSE</code> .
<code>lambda_method</code>	One of <code>"mc_exact"</code> , <code>"mc_gaussian"</code> , <code>"analytical"</code> . See <code>lambda_pdb()</code> for details. The default <code>"mc_exact"</code> is a safe choice for small to moderate designs; for very large $n$ or $p$ , <code>"mc_gaussian"</code> matches <code>"mc_exact"</code> closely at a fraction of the cost (it amortises only one BLAS gemm, no family-specific residual draws), and <code>"analytical"</code> is closed-form.
<code>relax</code>	If TRUE, run an unpenalised refit on the selected support after the regularisation path (debiasing).
<code>mcp_gamma</code>	MCP concavity parameter when <code>penalty = "mcp"</code> . Default 3.0.

scad_a	SCAD concavity parameter when <code>penalty = "scad"</code> . Default 3.7.
lambda	Optional user-supplied regularisation parameter. When <code>NULL</code> , <code>lambda_pdb()</code> is used to select it automatically. Providing <code>lambda</code> avoids recomputing the PDB calibration, which is useful for example when fitting several penalties (" <code>lasso</code> ", " <code>scad</code> ", " <code>mcp</code> ") on the same dataset, since the PDB choice of $\lambda$ does not depend on the penalty itself.
lambda_alpha	Nominal level for PDB. See <code>lambda_pdb()</code> .
lambda_n_simu	Number of Monte Carlo draws for PDB. See <code>lambda_pdb()</code> .
tol	FISTA convergence tolerance at the final path step.
path_length	Number of points in the warm-start regularisation path running from $\lambda_{\max}$ down to $\lambda^{\text{PDB}}$ . Default 10.
maxit	Maximum number of iterations for FISTA if convergence is not yet reached.

## Details

Minimises the objective function

$$\hat{\beta} = \arg \min_{\beta} L(\beta) + \lambda_{\alpha}^{\text{PDB}} \text{pen}(\beta) \quad \text{with} \quad L(\beta) = \phi(\ell_n(g(\beta))),$$

where the regularisation parameter is selected automatically using the Pivotal Detection Boundary (PDB) principle implemented in `lambda_pdb()`. The PDB choice is not just a substitute for cross-validation: by calibrating  $\lambda$  against a pivotal null-distribution statistic rather than out-of-sample prediction error, it yields sharper support recovery - fewer false positives on noise variables and more accurate identification of the true non-zero coefficients. Here,  $\phi$  and  $g$  are family-specific transformations chosen so that the gradient at the null has a distribution free of the nuisance parameter, which is precisely what makes the use of  $\lambda_{\alpha}^{\text{PDB}}$  valid. Optimisation is performed using a warm-started FISTA regularisation path.

### Details on family-specific losses:

"gaussian" Uses  $\phi(\cdot) = \sqrt{\cdot}$  and  $g(\cdot) = \text{Id}$ , resulting in the square-root lasso objective

$$L(\beta) = \frac{\|y - X\beta\|_2}{\sqrt{n}}.$$

"binomial" Uses a variance-stabilised transformation of the Bernoulli likelihood. With  $\phi(\cdot) = \text{Id}$  and the logistic link  $g(\eta_i) = (1 + e^{-\eta_i})^{-1}$ , the loss is

$$L(\beta) = \frac{1}{n} \sum_{i=1}^n \left( 2y_i \sqrt{\frac{1-g(\eta_i)}{g(\eta_i)}} + 2(1-y_i) \sqrt{\frac{g(\eta_i)}{1-g(\eta_i)}} \right).$$

The classical logistic link is preserved; only the loss itself is modified to obtain a pivotal gradient.

"poisson" As for "binomial", uses a variance-stabilised version of the Poisson likelihood. With  $\phi(\cdot) = \text{Id}$  and  $g(\eta_i) = e^{\eta_i}$ , the loss is

$$L(\beta) = \frac{1}{n} \sum_{i=1}^n \left( \frac{2y_i}{\sqrt{g(\eta_i)}} + 2\sqrt{g(\eta_i)} \right).$$

The standard log link is kept unchanged; the pivotality property is obtained through the modified loss rather than through the link function.

"exponential" Uses the standard Exponential negative log-likelihood together with the classical log link  $g(\eta_i) = e^{\eta_i}$ . The loss is

$$L(\beta) = \frac{1}{n} \sum_{i=1}^n \left( \log(g(\eta_i)) + \frac{y_i}{g(\eta_i)} \right).$$

No variance-stabilising modification is required.

"gumbel" Uses an exponentially stabilised Gumbel negative log-likelihood. With  $\phi(\cdot) = \exp(\cdot)$  and identity link  $g(\eta_i) = \eta_i$ , the loss is based on the Gumbel log-likelihood

$$\ell(\beta, \sigma) = \log(\sigma) + \frac{1}{n} \sum_{i=1}^n (z_i + e^{-z_i}), \quad z_i = \frac{y_i - \eta_i}{\sigma}.$$

The optimisation objective becomes

$$L(\beta, \sigma) = \exp(\ell(\beta, \sigma)).$$

The scale parameter  $\sigma$  is re-estimated internally by profile maximum likelihood at each optimisation step.

"cox" Uses a square-root-transformed Cox partial log-likelihood. With  $\phi(\cdot) = \sqrt{\cdot}$  and identity link  $g(\eta_i) = \eta_i$ , the underlying partial likelihood is

$$\ell(\beta) = -\frac{1}{n} \left[ \sum_{i=1}^n \delta_i \eta_i - \sum_{i=1}^n \delta_i \log \left( \sum_{j \in R_i} e^{\eta_j} \right) \right],$$

where  $\delta_i$  is the event indicator and  $R_i$  is the Cox risk set at time  $t_i$ . The final objective is

$$L(\beta) = \sqrt{\ell(\beta)}.$$

## Value

An object of class `c("pic.<family>", "pic")`.

<code>beta</code>	Fitted coefficients (length p).
<code>intercept</code>	Fitted intercept or NULL.
<code>df</code>	The number of nonzero coefficients.
<code>selected</code>	Indices of the nonzero coefficients.
<code>lambda</code>	$\lambda$ used during training (PDB or user-supplied).
<code>family</code>	The family object used.
<code>penalty</code>	The penalty object used.
<code>lambda_pdb</code>	Result from <code>lambda_pdb()</code> if $\lambda$ is not user-supplied.
<code>n_samples</code>	The number of observations in the training set.
<code>n_features</code>	The number of variables in the training set.

For family = "cox", the fit additionally carries:

<code>baseline_cumulative_hazard</code>	Estimated Breslow baseline cumulative hazard function.
<code>baseline_survival</code>	Estimated baseline survival function.
<code>unique_times</code>	Sorted unique event times used in the baseline estimation.
<code>censoring_rate</code>	Percentage of censored observations in the training set.

**Examples**

```

data(QuickStartExample)
X <- QuickStartExample$X
y <- QuickStartExample$y
fit <- pic(X, y, family = "gaussian", penalty = "scad")
fit$beta
fit$selected

```

---

pic\_families

*GLM families for pic — descriptor layer.*


---

**Description**

Each family is a thin descriptor carrying:

- name - used by the dispatchers to route to the C++ implementation.
- g - mean-function link (object with name and callable fn).  $g\eta$  is applied at predict time to obtain the mean response.
- phi - variance-stabilising transform (object with name). Purely informational on the R side; the actual stabilisation happens inside the C++ math.

**Details**

Typing `fit$family` gives a one-glance summary of the model's family (see `print.pic.family()`). The actual loss / gradient math lives in C++:

- `src/family_*.cpp` — Gaussian, Binomial, Poisson, Exponential, Gumbel.
- `src/cox.cpp` — Cox.

---

pic\_methods

*S3 methods for fitted pic objects.*


---

**Description**

The fitted object has class `c("pic.<family>", "pic")` — methods dispatch on the second class for shared behaviour and on the first class for family-specific overrides.

pic\_penalties

*Sparsity-inducing penalties for pic.***Description**

Three penalties are supported, identified by lowercase name to match the C++ registry. Each penalty enters the `pic()` objective as

$$\text{pen}(\beta) = \sum_{j=1}^p p_{\lambda}(|\beta_j|),$$

where  $p_{\lambda}(\cdot)$  depends on the penalty.

**Details**

"lasso" L1 (soft-thresholding) penalty:

$$p_{\lambda}(|t|) = \lambda|t|.$$

Convex, gives the strongest shrinkage on large coefficients bias does not vanish as  $|t| \rightarrow \infty$ .

"scad" (**Smoothly Clipped Absolute Deviation, Fan & Li 2001**) Non-convex penalty with concavity parameter `scad_a` > 2 (default 3.7):

$$p'_{\lambda}(|t|) = \lambda \left\{ \mathbf{1}\{|t| \leq \lambda\} + \frac{(a\lambda - |t|)_+}{(a-1)\lambda} \mathbf{1}\{|t| > \lambda\} \right\}.$$

Behaves like the lasso for small  $|t|$ , then tapers off so large coefficients are barely penalised - yields nearly unbiased estimates on strong signals.

"mcp" (**Minimax Concave Penalty, Zhang 2010**) Non-convex penalty with concavity parameter `mcp_gamma` > 1 (default 3.0):

$$p'_{\lambda}(|t|) = \left( \lambda - \frac{|t|}{\gamma} \right)_+.$$

Similar motivation as SCAD but a smoother transition: starts at the lasso derivative for small  $|t|$  and tapers linearly to zero at  $|t| = \gamma\lambda$ .

The actual evaluation and proximal operators live in C++ (`src/penalty_*.cpp`). Larger `scad_a` / `mcp_gamma` make the penalty closer to the lasso; smaller values amplify the non-convexity (and the bias reduction on strong signals).

---

pic_plots	<i>Plot methods for pic fits and diagnostics.</i>
-----------	---

---

### Description

Entry points:

### Details

- `plot(fit)` - lollipop plot of the non-zero coefficients.
- `plot(fit$lambda_pdb)` - histogram of the PDB null distribution with a vertical line at the selected  $\hat{\lambda}$ .
- `plot_baseline()` - Cox-only: baseline cumulative hazard and baseline survival, two panels.
- `plot(phase_transition(...))` - recovery curves.
- `plot(pdb_asymptotic(...))` - null-distribution convergence panels.

---

pic_survival	<i>Survival utilities for the Cox family.</i>
--------------	---

---

### Description

Provides Harrell's C-index, the Breslow partial log-likelihood, baseline cumulative hazard / survival, and feature-effect curves.

---

plot.pic	<i>Horizontal lollipop plot of the non-zero coefficients of a pic fit.</i>
----------	--

---

### Description

One row per selected variable, sorted by descending absolute coefficient value (largest at the top). Each variable is drawn as a horizontal segment from zero to its fitted value.

### Usage

```
## S3 method for class 'pic'
plot(x, standardized = TRUE, max_features = NULL, ...)
```

### Arguments

x	A fitted pic object.
standardized	Logical; if TRUE, plot the standardised coefficients used internally during fitting. If FALSE, use the coefficients on the original scale of X. Default TRUE.
max_features	Optional cap on the number of features displayed (the strongest are kept).
...	Additional graphical parameters forwarded to <code>graphics::plot()</code> for the empty frame.

**Value**

Invisibly returns the plotted (named) coefficient vector.

---

plot.pic.lambda\_pdb     *Plot the PDB null distribution.*

---

**Description**

Histogram of the simulated null gradient-norm statistics, with a vertical line at the selected  $\hat{\lambda}$ .

**Usage**

```
## S3 method for class 'pic.lambda_pdb'
plot(x, breaks = 40L, ...)
```

**Arguments**

x	A pic.lambda_pdb object (typically fit\$lambda_pdb).
breaks	Number of histogram bins (default 40).
...	Unused; present for S3 method consistency.

**Value**

Invisibly returns x.

---

plot.pic.pdb\_asymptotic  
                                   *Plot of the PDB asymptotic behaviour.*

---

**Description**

Multi-panel histogram comparison of the simulated null gradient-norm statistic under the "mc\_exact" (light grey fill) and "mc\_gaussian" (dashed outline) selectors, one panel per n in n\_grid. Two vertical lines are added per panel:

- $\hat{\lambda}_\alpha^{\text{PDB}}$  - solid navy, the empirical (1 - alpha) quantile of "mc\_exact" (what pic would actually use).
- $\hat{\lambda}_{\text{analytical}}$  - dashed black, the Bonferroni closed-form bound.

**Usage**

```
## S3 method for class 'pic.pdb_asymptotic'
plot(x, breaks = 40L, ...)
```

**Arguments**

x	An object returned by <code>pdb_asymptotic()</code> .
breaks	Number of histogram bins (default 40).
...	Unused; present for S3 method consistency.

**Value**

Invisibly returns x.

---

`plot.pic.phase_transition`

*Phase-transition plot for a `pic.phase_transition` object.*

---

**Description**

Plots the chosen recovery metric as a function of the sparsity level  $s$ . Curves are distinguished by line type (grayscale ramp beyond five curves). When multiple  $(n, p)$  configurations are compared across several penalties, panels are laid out in a grid with at most three columns per row.

**Usage**

```
## S3 method for class 'pic.phase_transition'  
plot(x, metric = c("exact_recovery", "tpr", "fdr"), ...)
```

**Arguments**

x	An object returned by <code>phase_transition()</code> .
metric	One of "exact_recovery" (default), "tpr", "fdr".
...	Additional graphical parameters forwarded to <code>graphics::plot()</code> .

**Value**

Invisibly returns x.

---

plot\_baseline      *Plot Cox baseline cumulative hazard and baseline survival.*

---

### Description

Two-panel step plot for a fitted `pic.cox` model: the Breslow baseline cumulative hazard  $H_0(t)$  on top and the baseline survival  $S_0(t) = \exp(-H_0(t))$  below.

### Usage

```
plot_baseline(model)
```

### Arguments

model              A fitted `pic.cox` object.

### Value

Invisibly returns NULL.

---

plot\_survival\_curves      *Plot subject-specific Cox survival curves.*

---

### Description

Step-line visualisation of the output of `predict_survival_function()` or `feature_effects_on_survival()`: one survival curve per subject (or per feature value) on a common time grid. To keep curves distinguishable, each curve is drawn with its own line type and a sparse set of marker glyphs is overlaid at regular time points.

### Usage

```
plot_survival_curves(
  sf,
  subjects = NULL,
  max_subjects = 10L,
  labels = NULL,
  n_marks = 8L,
  main = "Individual survival curves",
  ...
)
```

**Arguments**

sf	A list as returned by <code>predict_survival_function()</code> or <code>feature_effects_on_survival()</code> , with components <code>time</code> (length <code>K</code> ) and <code>survival</code> (matrix <code>K × m</code> , one column per curve).
subjects	Optional integer vector selecting which columns of <code>sf\$survival</code> to plot. Defaults to all curves, capped to the first <code>max_subjects</code> for legibility.
max_subjects	Maximum number of curves drawn when <code>subjects</code> is <code>NULL</code> . Default 10.
labels	Optional character vector of labels used in the legend, one per plotted curve. Defaults to the column names of <code>sf\$survival</code> when set, otherwise "subject 1", "subject 2", etc.
n_marks	Number of marker glyphs overlaid on each curve to help distinguish them. Default 8. Set to 0 to disable.
main	Plot title.
...	Additional graphical parameters forwarded to <code>graphics::matplot()</code> .

**Value**

Invisibly returns `NULL`.

---

predict.pic	<i>Linear predictor / response prediction for a pic fit.</i>
-------------	--

---

**Description**

Linear predictor / response prediction for a pic fit.

**Usage**

```
## S3 method for class 'pic'
predict(object, newx, type = c("response", "link", "class"), ...)
```

**Arguments**

object	A fitted pic object.
newx	Matrix of new values at which predictions are to be made.
type	"link" (linear predictor) or "response" (default; family g-link applied).
...	Unused; present for S3 method consistency.

**Value**

A numeric vector.

---

predict\_survival\_function  
*Survival curves for new data from a fitted Cox pic model.*

---

**Description**

Survival curves for new data from a fitted Cox pic model.

**Usage**

```
predict_survival_function(object, newx)
```

**Arguments**

object	A fitted pic.cox model.
newx	New design matrix (rows = subjects).

**Value**

A list with time (length K) and survival (matrix K x m, one column per subject).

---

print.pic.coef      *Print a pic.coef table.*

---

**Description**

Pretty-prints the two-column coefficient table returned by `coef.pic()` without the integer row numbers that `print.data.frame` would otherwise add. The underlying object is still a `data.frame`, so any subsetting / column access works as usual.

**Usage**

```
## S3 method for class 'pic.coef'
print(x, ...)
```

**Arguments**

x	A pic.coef object.
...	Forwarded to <code>print.data.frame()</code> .

**Value**

Invisibly returns x.

---

print.pic.family	<i>Pretty-print a pic family descriptor.</i>
------------------	--

---

**Description**

Three-line summary showing the family name and its (g, phi) link pair.

**Usage**

```
## S3 method for class 'pic.family'  
print(x, ...)
```

**Arguments**

x	A pic.family object.
...	Ignored.

**Value**

Invisibly returns x.

---

print.pic.pdb_asymptotic	<i>Print PDB asymptotic diagnostic.</i>
--------------------------	---

---

**Description**

Print PDB asymptotic diagnostic.

**Usage**

```
## S3 method for class 'pic.pdb_asymptotic'  
print(x, ...)
```

**Arguments**

x	A pic.pdb_asymptotic object.
...	Unused; present for S3 method consistency.

**Value**

Invisibly returns x.

---

```
print.pic.phase_transition
      Print phase-transition analysis.
```

---

**Description**

Print phase-transition analysis.

**Usage**

```
## S3 method for class 'pic.phase_transition'
print(x, ...)
```

**Arguments**

x	A <code>pic.phase_transition</code> object.
...	Unused; present for S3 method consistency.

**Value**

Invisibly returns x.

---

```
QuickStartExample      Small Gaussian dataset for the introductory vignette.
```

---

**Description**

A synthetic Gaussian regression dataset used to illustrate `pic()` throughout the introductory vignette. It contains  $n = 100$  observations of  $p = 30$  predictors, of which only  $s = 5$  carry signal; the remaining 25 are pure noise.

**Usage**

```
QuickStartExample
```

**Format**

A list with two components:

X	Numeric matrix of dimension $100 \times 30$ with column names <code>gene_*</code> and <code>noise_*</code> .
y	Numeric vector of length 100.

**Details**

Column names are chosen to make the underlying support obvious at a glance:

- gene\_1, ..., gene\_5: the five active variables, whose non-zero coefficients are drawn uniformly in  $[0.5, 1.5]$  with random sign.
- noise\_1, ..., noise\_25: the remaining inactive variables, with true coefficient 0.

The columns are interleaved in random order; column names are the only indicator of which features are part of the true support.

The response is generated as  $y = X\beta + \varepsilon$  with  $\varepsilon \sim \mathcal{N}(0, 1)$ .

**Examples**

```
data(QuickStartExample)
fit <- pic(QuickStartExample$X, QuickStartExample$y)
fit$selected
```

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